Extrapolation between the nodes $\tau_{k-1}$, and $\tau_k$ was carried out from the two preceding points $\tau_{k-1}$ and $\tau_{k-2}$. The numerical solution of (2.19) for n = 1 is given in Fig. 1. As an example, let us discuss the case of the course of a zero-order reaction. The rate of the chemical reaction is equal to k exp (—E/RT) for $0 \le y \le 1$ and vanishes for $y > 1$. The solution of (2.19) is of the form

$$z(\tau) = \varepsilon[1 - (2/\pi)E(\sqrt{\tau})],$$

where E(x) is the complete elliptic integral. For $0 < \varepsilon \le \pi/(\pi - 2) = 2.752$, the ignition time $\tau^* = 1$, and $z(\tau^*) = \varepsilon\ [(\pi - 2)/\pi]$. For $\varepsilon < 2.752$, the chemical reaction ceases at the instant $\tau = \tau^+$ $[z(\tau^+) = 1]$, where $\tau^+$ is the root of the equation $1 = \varepsilon[1 - (2/\pi)E(\sqrt{\tau^+})]$.

LITERATURE CITED


1. Ya. B. Zel'dovich, "Contribution to ignition theory," Dokl. Akad. Nauk SSSR, 150, No. 2 (1963).
2. K. G. Shkadinskii and V. V. Barzykin, "Regularities of the ignition of gases by a heated surface with diffusion and hydrodynamics taken into account," Fiz. Goreniya Vzryva, No. 2 (1968).
3. A. G. Merzhanov, B. I. Khaikin, and K. G. Shkadinskii, "Establishment of steady-state propagation of a flame upon the ignition of a gas by a heated surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1969).
4. A. G. Merzhanov and A. E. Averson, "Contemporary state of thermal ignition theory," Preprint Inst. Khim. Fiz. Akad. Nauk SSSR, Moscow (1970).
5. A. Linian and F. A. Williams, "Theory of ignition of a reaction solid by constant energy flux combustion," Sci. Technol., 3, 91 (1971).
6. A. Linian and F. A. Williams, "Radiant ignition of a reactive solid with in-depth absorption," Combust. Flame, 18, 85 (1972).
7. V. N. Vilyunov and R. S. Gol'dman, "Application of the method of spliced asymptotic expansions to one problem of ignition," in: Data of the Fourth Conference on Mathematics and Mechanics [in Russian], Izd. Tomsk. Univ., Tomsk (1974).
8. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press (1964).
9. J. D. Cole, Perturbation Methods in Applied Mathematics, Blaisdell.


# NUMERICAL STUDY OF THE EFFECT OF SURFACE THERMAL CONDITIONS ON FLOW IN THE BASE REGION OF A BODY OF FINITE DIMENSIONS


A. V. Babakov and L. I. Severinov



UDC 533.6.011


The results of the present work are obtained using the method of "fluxes" [1], which has properties of conservativity with respect to mass, momentum, and total energy. A characteristic feature of the method is the asymmetrical approximation of the convective terms.

It is assumed that the gas is Newtonian and perfect, has constant specific heat capacities, the coefficient of viscosity μ depends on the temperature in accordance with the law $\mu \sim T^{\omega}$ ($\omega$ = const), and the Prandtl number Pr is constant. Moreover, the Stokes hypothesis of the equality of the pressure and of the arithmetic mean of the three principal stresses with the opposite sign is satisfied. The calculations were carried out in the following coordinate system: the x axis is directed along the surface of the sphere, the y axis along the local normal to it, and the origin is placed at the leading critical point. We introduce the following notation: u and v are the velocity components along x and y; p, ρ, and T


Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 73-76, January-February, 1977. Original article submitted January 19, 1976.

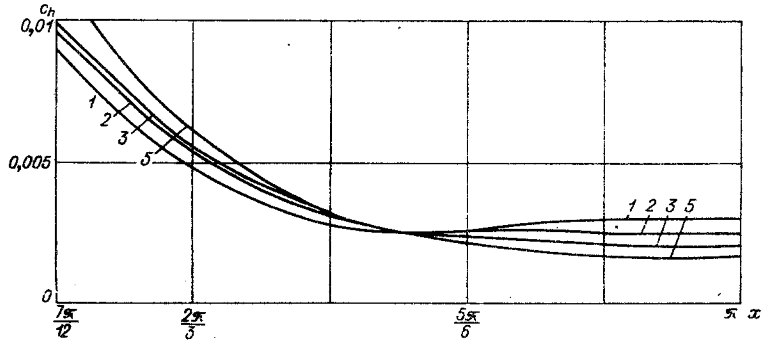


Fig. 1

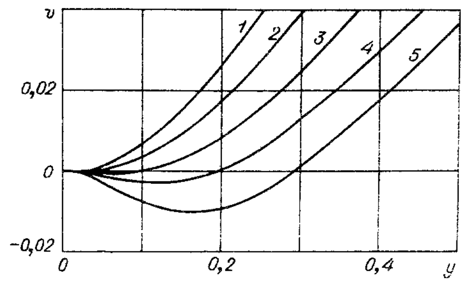


Fig. 2

are the pressure, density, and temperature, respectively; $\gamma$ is the ratio of specific heat capacities. The dimensionless u, v, p, $\rho$, T, and $\mu$ are obtained by dividing the dimensional variables by $V_\infty$, $V_\infty$, $\rho_\infty V^2_\infty$, $\rho_\infty$, $T_\infty$, and $\mu_\infty$, where $V_\infty$ is the modulus of the velocity vector in the undisturbed stream. The linear dimensions are normalized to the radius R of the sphere over which the flow occurs.

The results of the numerical study were obtained with the following values of the determining parameters: $M_\infty = 20$, $Pr = 0.72$, $Re_\infty = \rho_\infty V_\infty R/\mu_\infty = 1500$, $\gamma = 1.4$, $\omega = 0.5$. As the parameter characterizing the temperature of the surface we take the quantity $k = T_w/T_0$, where $T_w$ is the temperature of the surface; $T_0$ is the temperature of the adiabatically stagnated gas.

The region of integration is the spherical layer included between the surface of the sphere over which the flow occurs and an outer spherical boundary of radius $R_b$. The calculations were conducted on a grid with 30 × 30 cells with constant steps along the coordinates for $R_b = 1.75$. The solution was obtained in the entire region of integration at once, although here we present the results pertaining only to the base region where the effect of k on the character of the flow is the most important.

Systematic calculations conducted earlier [2] showed the possibility of the numerical study of flows with the values of the determining parameters and the grid chaacteristics indicated above.

The systematic calculations of [2] were conducted for the smallest value of k = 0.05 used in the present calculations, for which the calculation is the most difficult from the computational point of view.

Flows with values of k equal to 0.5 (variant 1), 0.375 (2), 0.25 (3), 0.15 (4), and 0.05 (5) were studied. We note that for these values of k the numerical solution has a separation character, except for k = 0.5, where the separation may only be incipient.

In Fig. 1 (curves 1-3 and 5 correspond to the variants of the values of k) we show the distribution over the surface of the sphere of the local coefficient of heat transfer

$$c_h = (\lambda \partial T/\partial y)_w/\rho_\infty V_\infty(H_\infty - H_w),$$

where $\lambda$ is the coefficient of thermal conductivity; H is the total enthalpy. In the region of $x \gtrsim 5\pi/6$ the value of $c_h$ increases with an increase in k for $k \geq 0.05$, whereas it decreases with an increase in k for $\pi/2 < x \lesssim 5\pi/6$.

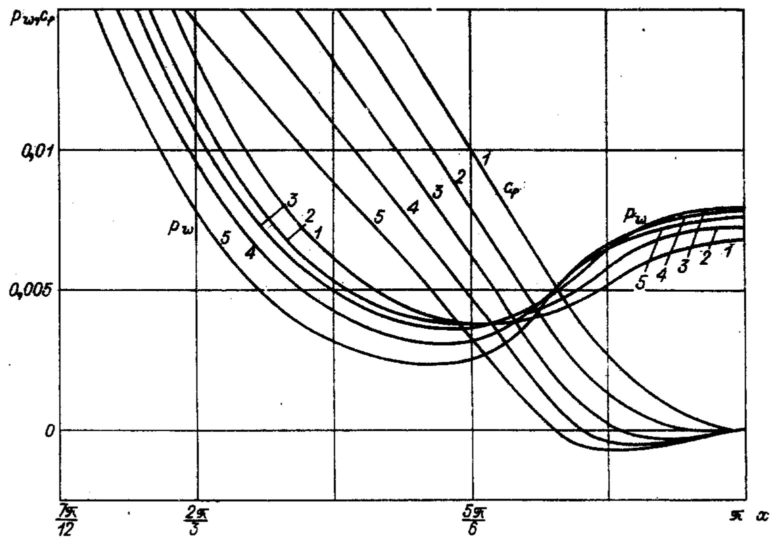


Fig. 3

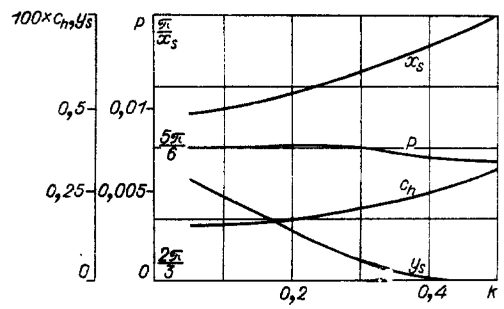


Fig. 4

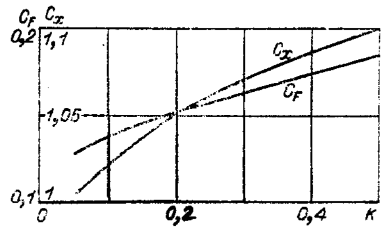


Fig. 5

Graphs of the velocity v normal to the surface of the body along the rear axis of symmetry are presented in Fig. 2. It is seen from them that a decrease in k causes an increase in the length of the region of return circulation behind the sphere.

The distributions over the surface of the sphere of the pressure p and the coefficient of surface friction

$$c_f = 2\,(\mu\partial u/\partial y)_w/\rho_\infty V_\infty^2$$

are shown in Fig. 3. The behavior of $c_f$ indicates that a decrease in k leads to movement of the point of separation up along the surface of the sphere. The separation occurs in the region of a positive pressure gradient, which is in accordance with the Prandtl concept of separation.

With an increase in x the pressure in the base region goes onto a so-called pressure plateau. There is a weakly expressed pressure maximum at the base critical point for $k \approx 0.25$.

The effect of the parameter k (or of the surface temperature, which is the same) on the coordinate $x_s$ of the point of separation of the stream, the length $y_s$ of the region of return circulation along the rear axis of symmetry, and on the values of $p_w$ and $c_h$ at the rear critical point is shown in Fig. 4. It is seen that the pressure at the rear critical point is practically independent of the surface temperature (at least in the investigated range of variation of k). The position of the point of separation and the length of the region of return circulation vary markedly with a decrease in the surface temperature: for $k = 0.5$ the flow has a nonseparation character; cooling of the surface of the body leads to the appearance of separation and an increase in the region of return circulation.

The dependences of the coefficient of total resistance $C_x$ of the sphere and of the resistance $C_F$ due to the presence of viscosity are shown in Fig. 5. The total resistance and the frictional resistance decrease with cooling of the surface of the sphere.

LITERATURE CITED

1. O. M. Belotserkovskii and L. I. Severinov, "The conservative method of 'fluxes' and calculation of the flow of a viscous heat-conducting gas over a body of finite dimensions," Zh. Vychisl. Mat. Mat. Fiz., 13, No. 2, 385-397 (1973).
2. A. V. Babakov, O. M. Belotserkovskii, and L. I. Severinov, "Numerical study of the flow of a viscous heat-conducting gas over a blunt body of finite dimensions," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 112-123 (1975).

# STRUCTURE OF A MULTICOMPONENT BOUNDARY LAYER IN MODELS UNDERGOING BREAKDOWN

É. B. Georg and M. I. Yakushin UDC 532.526

§1. Experiments were carried out in a high-temperature gasdynamic apparatus, using an inductive high-frequency discharge in an air flow as the gas heater [1]. The arrangement of the apparatus is illustrated in Fig. 1, where 1 is a radio-frequency generator, 2 is the test chamber separating the discharge channel and heating inductor 3 from the surrounding atmosphere, 4 is a two-component locator, 5 is the test model, 6 is the extraction tube (exhaust pipe), 7 is a centrifugal blower with a rate of delivery capable of smooth variation, 8 is a set of flowmeters in the gas-supply tract, 9 is a screw-type air compressor, and 10 is a set of inert-gas cylinders. The apparatus was usually operated in air from a compressor creating an excess pressure of 0.25 atm. In order to facilitate the starting of the apparatus, argon was employed, the system being connected to gas cylinders. As source of electrical energy for the inductive discharge, a radio-frequency tube generator operating at a frequency of 17 MHz was used. The maximum oscillatory power consumed was 50 kW. The cylindrical discharge channel comprised a quartz tube carrying a double-wound copper solenoid, playing the part of inductor and heating element.

The flow of gas passed into the discharge channel from a preliminary chamber, in which it was given a spiral—translational motion. The preliminary chamber contained a profiled annular space to which the gas was fed tangentially. Depending on the swirling intensity of the incident gas, three gasdynamic modes of discharge initiation were encountered. The best situation was that in which the discharge region was filled with hot gas, the discharge ignited from the wall, and the process continued for an unlimited time. For weak swirling of the gas, the discharge sticks to the walls, and this leads to thermal decomposition of the quartz; for intense swirling the discharge is drawn upward along the flow and is ultimately blown away by flows of cold gas. The results described in this paper were obtained under optimum gasdynamic discharge conditions. A characteristic feature of the process lies in the fact that the lateral leading edge of the discharge is inclined to the incident flow in such a way that the cold gas passes into the "flame" along the normal at a rate of ~10 cm/sec and on being heated is accelerated to a velocity of ~30 m/sec in the axial direction.

It is well known that the mechanism for producing inductive plasma involves the existence of induced voltage pulses, which may be the reason for the appearance of pulsations in the plasma. Special experiments showed that the time interval between the voltage pulses ($t \sim 10^{-8}$ sec) was several orders of magnitude shorter than the plasma attenuation time ($t \sim 10^{-1}$ sec). An analysis of high-speed motion pictures indicated the absence of intensity pul-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 76-80, January-February, 1977. Original article submitted November 18, 1975.